

1

(1)  $\frac{\Delta y}{\Delta x} = \frac{f(3)-f(2)}{3-2} = (2 \cdot 3^2 - 3 + 5) - (2 \cdot 2^2 - 2 + 5) = 20 - 11 = 9$  //

(2)  $y = f'(1)(x-1) = -2(x-1) \therefore y = -2x + 2$  //

(3)  $f'(x) = 3x^2 + 2kx + 2 = 0$  の判別式  $D/4 \leq 0$  かつ  
 $k^2 - 6 \leq 0 \iff -\sqrt{6} \leq k \leq \sqrt{6}$  //

(4)  $F(x) = ax^2 + bx + c$  とし

$F(x) = \int (4x+2) dx = 2x^2 + 2x + C \quad F(0) = 1$  かつ  $C = 1 \therefore a = b = 2, c = 1$  //

(5)  $\int_a^x f(x) dx = x^2 + 2x - 3$  の両辺を  $x$  で微分すると  
 $f(x) = 2x + 2$  //

よって  $\int_a^x (2t+2) dt = \left[ t^2 + 2t \right]_a^x = x^2 + 2x - a^2 - 2a$

$a^2 + 2a = 3 \quad (a+3)(a-1) = 0 \quad \therefore a = -3, a = 1$  //

(6)  $f(x) = x + \int_0^2 f(x) dx$

$a = \int_0^2 f(x) dx$  とおくと  $f(x) = x + a$  かつ  $f(x) = x + a$  である

$\therefore a = \int_0^2 (x+a) dx = \left[ \frac{1}{2}x^2 + ax \right]_0^2 = 2 + 2a \quad a = 2 + 2a \rightarrow a = -2$   
 $\therefore f(x) = x - 2$  //

2  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^2 + 3(x+h) - (x^2 + 3x)}{h}$   
 $= \lim_{h \rightarrow 0} \frac{(2x+h+3)h}{h} = \lim_{h \rightarrow 0} (2x+h+3) = 2x+3$  //

3  $y' = 3x^2 - 12x + 9$

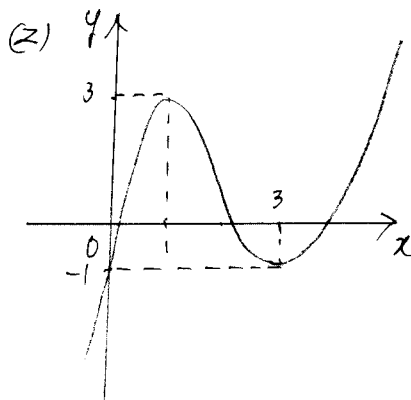
(1)  $y' = 0$  かつ  $x^2 - 4x + 3 = 0$   
 $(x-1)(x-3) = 0$   
 $x = 1, x = 3$

$x$	...	1	...	3	...
$y'$	+	0	-	0	+
$y$	$\nearrow$	3	$\searrow$	-1	$\nearrow$

増減表より

極大値 3 ( $x=1$ )

極小値 -1 ( $x=3$ ) //

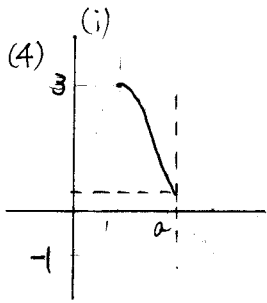


(3)  $y = x^3 - 6x^2 + 9x - 1$

$y = m$  との交点の数が 3 個となる

$m$  の値の範囲は  $-1 < m < 3$  //

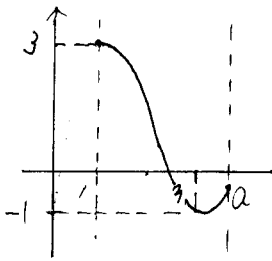
3



(i)  $1 < a \leq 3$  and  $a \leq 3$

$M = 3$  ( $x=1$ )  
 $m = a^3 - 6a^2 + 9a - 1$  ( $x=a$ )

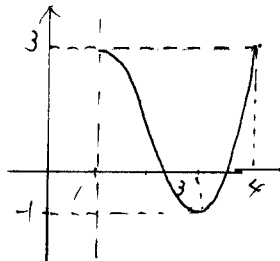
(ii)



(ii)  $3 < a < 4$  and  $a \leq 3$

$M = 3$  ( $x=1$ )  
 $m = -1$  ( $x=3$ )

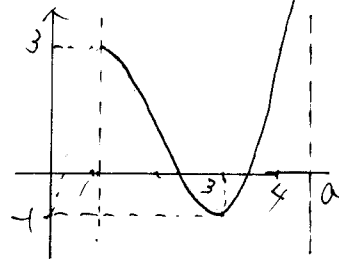
(iii)



(iii)  $a = 4$  and  $a \leq 3$

$M = 3$  ( $x=1, 4$ )  
 $m = -1$  ( $x=3$ )

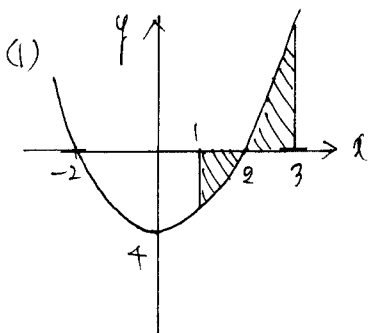
(iv)



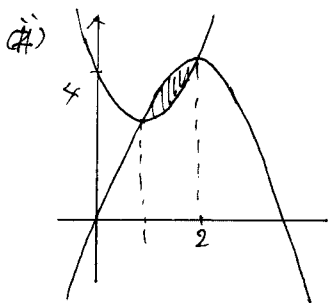
(iv)  $4 < a$  and  $a \leq 3$

$M = a^3 - 6a^2 + 9a - 1$  ( $x=a$ )  
 $m = -1$  ( $x=3$ )

4



(i)  $S = -\int_1^2 (x^2 - 4x + 4) dx + \int_2^3 (x^2 - 4x + 4) dx$   
 $= \left[ -\frac{1}{3}x^3 + 2x^2 - 4x \right]_1^2 + \left[ \frac{1}{3}x^3 - 2x^2 + 4x \right]_2^3$   
 $= \left( -\frac{8}{3} + 8 - 4 \right) - \left( -\frac{1}{3} + 2 - 4 \right) + \left( \frac{27}{3} - 12 + 12 \right) - \left( \frac{8}{3} - 8 + 8 \right)$   
 $= \frac{27 + 1 - 16}{3} + 8 - 4 - 12 + 8 = \underline{\underline{4}}$



$x^2 - 2x + 4 = -x^2 + 4x$   
 $2x^2 - 6x + 4 = 0$   
 $2(x-1)(x-2) = 0$   
 $x = 1, 2$

$S = \int_1^2 (-x^2 + 4x) - (-x^2 + 2x + 4) dx$   
 $= -2 \int_1^2 (x-1)(x-2) dx$   
 $= \frac{2}{6} (2-1)^3 = \underline{\underline{\frac{1}{3}}}$

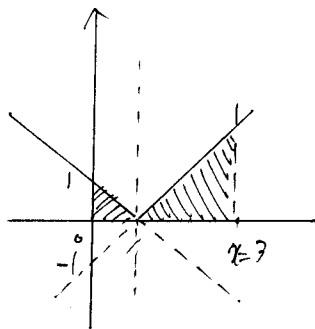
(2)  $y = |x-1|$

(i)  $x-1 \geq 0$  i.e.  $1 \leq x \leq 3$

$y = x-1$

(ii)  $x-1 < 0$  i.e.  $x < 1$  and  $x \leq 3$

$y = -x+1$



$\int_0^3 |x-1| dx$   
 $= \int_0^1 (-x+1) dx + \int_1^3 (x-1) dx$   
 $= \left[ -\frac{1}{2}x^2 + x \right]_0^1 + \left[ \frac{1}{2}x^2 - x \right]_1^3$   
 $= \frac{1}{2} + \left( \frac{3}{2} - \frac{1}{2} \right) = \underline{\underline{\frac{5}{2}}}$

5

(1)  $y = f(x) = x^2 + x + 1$  is tangent to ... ①

$f(x) = 2x + 1$  is tangent to ...

接点  $(t, t^2 + t + 1)$  is tangent to ...

接線は

$y - (t^2 + t + 1) = (2t + 1)(x - t)$

$y = (2t + 1)x - t^2 + 1$  ... ②

∴  $(0, 0)$  is on the line  
 $t^2 = 1 \rightarrow t = \pm 1$

i)  $t = 1$  and  $t \leq 3$  接点  $(1, 3)$

∴ 接線は  $y = 3x$

ii)  $t = -1$  and  $t \leq 3$  接点  $(-1, 1)$

∴ 接線は  $y = -x$

5 (2)

$$S = \int_{-1}^0 \{(x^2 + x + 1) - (-x)\} dx + \int_0^1 \{(x^2 + x + 1) - 3x\} dx$$

$$= \int_{-1}^0 (x^2 + 2x + 1) dx + \int_0^1 (x^2 - 2x + 1) dx$$

$$= \left[ \frac{1}{3}x^3 + x^2 + x \right]_{-1}^0 + \left[ \frac{1}{3}x^3 - x^2 + x \right]_0^1$$

$$= \left( \frac{1}{3} - 1 + 1 \right) + \left( \frac{1}{3} - 1 + 1 \right) = \frac{2}{3} //$$

<公式>

$$S = \frac{|a|}{12} (\beta - \alpha)^3$$

これを用いて

$$\frac{1}{12} \{1 - (-1)\}^3 = \frac{8}{12} = \frac{2}{3} \quad \varepsilon + \delta //$$

6 C:  $y = -\frac{1}{2}x^2 + 2x$

(1)  $-\frac{1}{2}x^2 + 2x = 0$   
 $x(x-4) = 0$   
 $x = 0, 4$

$$S = \int_0^4 \left(-\frac{1}{2}x^2 + 2x\right) dx$$

$$= -\frac{1}{2} \int_0^4 x(x-4) dx$$

$$= -\frac{1}{2} \cdot \left\{ -\frac{1}{6} (4-0)^3 \right\} = \frac{4^3}{12} = \frac{16}{3} //$$

(2)

$$mx = -\frac{1}{2}x^2 + 2x$$

$$x^2 + (2m-4)x = 0$$

$$x\{x + 2(m-2)\} = 0$$

$$x = 0, x = 4 - 2m$$

$$S_1 = \frac{1}{2} S = \frac{1}{2} \times \frac{16}{3} = \frac{8}{3} //$$

$$\int_0^{4-2m} \left\{ \left(-\frac{1}{2}x^2 + 2x\right) - mx \right\} dx$$

$$= -\frac{1}{2} \int_0^{4-2m} x(x + 2m - 4) dx$$

$$= \frac{1}{2} \cdot \frac{1}{6} (4-2m)^3 = \frac{8}{12} (2-m)^3 = \frac{8}{3} //$$

$(2-m)^3 = 4$   
 $2-m = \sqrt[3]{4}$   
 $0 < m < 2 //$   
 $m = 2 - \sqrt[3]{4} //$

7

高さ  $x$ , 半径  $r$ . 体積  $V$  を求めよ.

$$r^2 + 9^2 = x^2 \quad \therefore r^2 = x^2 - 81$$

$$\therefore V = \frac{1}{3} \times \pi \cdot r^2 \cdot x = -\frac{1}{3} \pi x^3 + 27\pi x$$

$$V' = -\pi x^2 + 27\pi \quad V' = 0 \quad \therefore -\pi(x-3\sqrt{3})(x+3\sqrt{3}) = 0 \rightarrow x = \pm 3\sqrt{3}$$

$\therefore 0 < x < 9$  の増減表を作成可能

$x$	0	...	$3\sqrt{3}$	...	9
$V'$	/	+	0	-	/
$V$	/		$54\sqrt{3}\pi$		/

最大値  $x = 3\sqrt{3}$  のとき  $V = -\frac{1}{3}\pi \cdot 27 \cdot 3\sqrt{3} + 27\pi \cdot 3\sqrt{3} = 54\pi\sqrt{3}$

$\therefore$  体積の最大値は  $54\pi\sqrt{3}$

高さ  $3\sqrt{3}$ , 体積  $54\sqrt{3}\pi //$