

1

(1) $y' = \frac{-(x^2-1)'}{(x^2-1)^2} = -\frac{2x}{(x^2-1)^2}$ (2) $y' = \frac{2}{3} x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}}$

(3) $y' = 6(2x^3+5)^5 \cdot (2x^3+3)' = 36x^2(2x^3+3)^5$ (4) $y' = \log x + x \cdot \frac{1}{x} - 1 = \log x$

(5) $y' = \cos(3x+2) \cdot (3x+2)' = 3\cos(3x+2)$

(6) $y' = (-3x+2)' e^{-3x+2} = -3e^{-3x+2}$

(7) $y' = \frac{(1+x^2)'(1-x^2) - (1-x^2)'(1+x^2)}{(1-x^2)^2} = \frac{2x(1-x^2) + 2x(1+x^2)}{(1-x^2)^2} = \frac{4x}{(1-x^2)^2}$

(8) $y' = \frac{1}{2} (2x^2+3)^{-\frac{1}{2}} \cdot (2x^2+3)' = \frac{2x}{\sqrt{2x^2+3}}$

* (9) $y = -\frac{\sin x}{\cos x - 1} = \frac{2\sin \frac{x}{2} \cos \frac{x}{2}}{2\sin^2 \frac{x}{2}} = \frac{\cos \frac{x}{2}}{\sin \frac{x}{2}} = \frac{1}{\tan \frac{x}{2}}$ (注) $(\tan \theta)' = \frac{1}{\cos^2 \theta}$
 $\therefore y' = -\frac{1}{\sin^2 \frac{x}{2}} \cdot (\frac{x}{2})' = -\frac{1}{2\sin^2 \frac{x}{2}}$ $(\frac{1}{\tan \theta})' = -\frac{1}{\sin^2 \theta}$

(10) $y' = \frac{(x^2+3)'}{(x^2+3)\log 2} = \frac{2x}{(x^2+3)\log 2}$ (11) $y' = \frac{(\log x)'}{\log x} = \frac{1}{x \log x}$

(12) 両辺に対数をとる $\log y = \sin x \cdot \log x$
 $\therefore \frac{y'}{y} = \cos x \log x + (\sin x) \left(\frac{1}{x}\right) \rightarrow y' = (\cos x \log x + \frac{\sin x}{x}) \cdot x^{\sin x} \leftarrow \text{両辺を } x^{\sin x} \text{ で } \times \text{ する}$
 $= (x \cos x \log x + \sin x) \cdot x^{\sin x - 1}$

2

(1) $\frac{x^2}{4} + \frac{y^2}{9} = 1 \leftrightarrow 9x^2 + 4y^2 = 36 \leftrightarrow 18x + 8y \cdot y' = 0 \leftrightarrow y' = -\frac{9x}{4y}$

(2) $\frac{dx}{d\theta} = 1 - \cos \theta, \frac{dy}{d\theta} = \sin \theta, \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\sin \theta}{1 - \cos \theta}$

(3) $y = e^{2x} \rightarrow y' = 2e^{2x} \rightarrow y'' = 2^2 e^{2x} \rightarrow \dots \rightarrow y^{(n)} = 2^n e^{2x}$

3

(1) $y = f(x) = \log x \rightarrow f'(x) = \frac{1}{x} \rightarrow f'(e) = \frac{1}{e} = e$
 A $(\frac{1}{e}, -1)$ における接線は $y - (-1) = e(x - \frac{1}{e}) \rightarrow y = ex - 2$
 " 法線は $y - (-1) = -\frac{1}{e}(x - \frac{1}{e}) \rightarrow y = -\frac{1}{e}x + \frac{1}{e^2} - 1$

(2) $2x^2 - 3y^2 = 6 \rightarrow 4x - 6y \cdot y' = 0 \rightarrow y' = \frac{2x}{3y} = \frac{2 \times 3}{3 \times 2} = 1$
 $\therefore y - 2 = 1 \times (x - 3) \rightarrow y = x - 1$ $\uparrow (x, y) = (3, 2)$ 点から

(3) $y = f(x) = \sqrt{x} \rightarrow f'(x) = \frac{1}{2\sqrt{x}}$

(1) $f'(x) = \frac{1}{2}$ かつ $\frac{1}{2\sqrt{x}} = \frac{1}{2} \rightarrow x = 1 \rightarrow$ 接点 $(1, 1)$ 点から
 \therefore 点における接線は $y - 1 = \frac{1}{2}(x - 1) \rightarrow y = \frac{1}{2}x + \frac{1}{2}$

(2) 接点 (t, \sqrt{t}) 点から \therefore 点における接線は
 $y - \sqrt{t} = \frac{1}{2\sqrt{t}}(x - t)$ \therefore 点 $(-1, 0)$ 点から $-\sqrt{t} = \frac{1}{2\sqrt{t}}(-1 - t)$ (此時) $t = 1$

(3) 点 $(1, \frac{1}{2})$ 点から $y = \frac{1}{2}x + \frac{1}{2}$

4 $f(x) = \log x$ であり、平均値の定理より

$$\frac{\log b - \log a}{b - a} = f'(c) \quad (a < c < b) \text{ を満たす } c \text{ の少なくとも一つが存在する。}$$

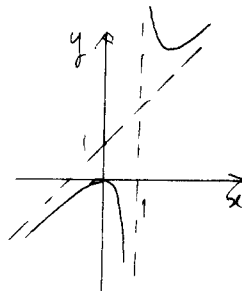
又 $f(x) = \frac{1}{x}$ より $f'(c) = \frac{1}{c} \rightarrow \frac{1}{b} < \frac{1}{c} < \frac{1}{a}$ となる。

$$\frac{1}{b} < \frac{\log b - \log a}{b - a} < \frac{1}{a} \rightarrow \frac{b-a}{b} < \log b - \log a < \frac{b-a}{a}$$

$$\therefore 1 - \frac{a}{b} < \log b - \log a < \frac{b}{a} - 1 \quad \text{となる。}$$

5 (1) $f(x) = \frac{x^2}{x-1} = x + 1 + \frac{1}{x-1}$

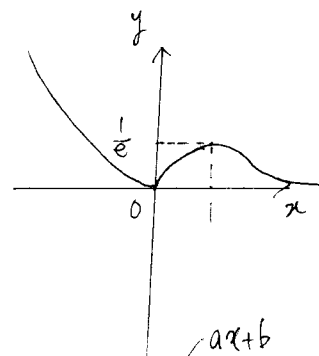
$$\begin{cases} \lim_{x \rightarrow 1-0} f(x) = -\infty & \lim_{x \rightarrow 1+0} f(x) = +\infty \\ \lim_{x \rightarrow \infty} f(x) = \infty & \lim_{x \rightarrow -\infty} f(x) = -\infty \end{cases}$$



(2) $f(x) = |x|e^{-x}$

i) $x \geq 0$ のとき $f(x) = xe^{-x} \therefore \lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{x}{e^x} = 0$

ii) $x < 0$ のとき $f(x) = -x \cdot e^{-x} \lim_{x \rightarrow -\infty} f(x) = \lim_{t \rightarrow \infty} t \cdot e^t = \infty$



6 $f(x) = \frac{ax+b}{x^2+2} \rightarrow f'(x) = \frac{(ax+b)'(x^2+2) - (ax+b)(x^2+2)'}{(x^2+2)^2} = \frac{a(x^2+2) - 2x(ax+b)}{(x^2+2)^2}$

$f(x)$ の $x=1$ での極値値 1 を与える

$$f(1) = 1 \text{ より } \frac{a+b}{3} = 1 \rightarrow a+b = 3 \dots \textcircled{1} \quad f'(1) = 0 \text{ より } \frac{3a-2(a+b)}{3^2} = 0 \rightarrow a-2b = 0 \dots \textcircled{2}$$

$\textcircled{1}, \textcircled{2}$ より $a=2, b=1$

$f(x)$ における a, b の存在 $f'(x) = \frac{-2x^2 - 2x + 4}{(x^2+2)^2} = -\frac{2(x+2)(x-1)}{(x^2+2)^2}$

$f'(x) = 0$ より $x = -2, x = 1$. よって $f(x)$ の極小値は $f(-2) = \frac{-4+1}{4+2} = -\frac{1}{2}$

7 $f(x) = \sin x (1 + \cos x) \quad (0 \leq x \leq 2\pi)$
 $= \sin x + \sin x \cos x$
 $= \sin x + \frac{1}{2} \sin 2x$

$f'(x) = \cos x + \frac{1}{2} \cdot \cos 2x \cdot (2x)' = \cos x + \cos 2x = \cos x + 2\cos^2 x - 1$
 $= (2\cos x - 1)(\cos x + 1)$

$f'(x) = 0$ かつ $\cos x = \frac{1}{2}, \cos x = -1 \rightarrow x = \frac{\pi}{3}, \frac{5}{3}\pi, \pi$

$f''(x) = -\sin x - 2\sin 2x = -\sin x - 4\sin x \cos x = -\sin x (1 + 4\cos x)$

$f''(x) = 0$ かつ $\sin x = 0 \rightarrow x = 0, \pi, 2\pi$

∴ f(x) の条件... 増減表を作成すると、

| | | | | | | |
|-------|---|-----------------|-------|------------------|--------|---|
| x | 0 | $\frac{\pi}{3}$ | π | $\frac{5}{3}\pi$ | 2π | |
| f'(x) | | + | 0 | - | 0 | + |
| f(x) | 0 | ↗ | ↘ | ↘ | ↗ | 0 |

※ $f''(\pi) = 0$ であるが $x = \pi$ で極値ではない。
 ∴ 注意

$f(\frac{\pi}{3}) = \frac{\sqrt{3}}{2} (1 + \frac{1}{2}) = \frac{3\sqrt{3}}{4}$

$f(\frac{5}{3}\pi) = -\frac{\sqrt{3}}{2} (1 + \frac{1}{2}) = -\frac{3\sqrt{3}}{4}$

∴ 最小値 $-\frac{3\sqrt{3}}{4}$ ($x = \frac{5}{3}\pi$)

最大値 $\frac{3\sqrt{3}}{4}$ ($x = \frac{\pi}{3}$)

8 $y = ax^2 + b, y = \log x$

$y = \log x \rightarrow y' = \frac{1}{x}$ かつ

接線の方程式は $y - 1 = \frac{1}{e}(x - e)$

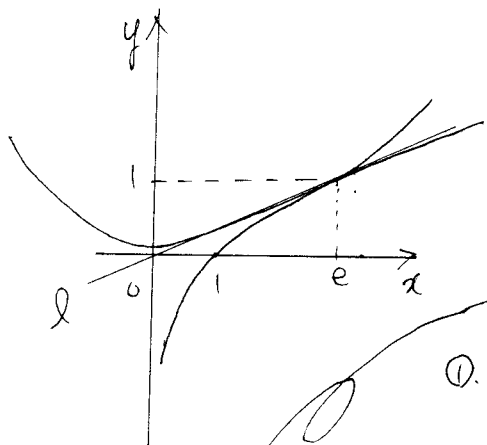
$y = \frac{1}{e}x \dots \textcircled{1}$

∴ $x = A(e, 1)$ における $y = ax^2 + b$ の接線は

$\frac{y+1}{2} = a \cdot e \cdot x + b \rightarrow y = 2ae x + (2b-1) \dots \textcircled{2}$

①, ② 比較して整理すると

$\begin{cases} \frac{1}{e} = 2ae \\ 2b-1 = 0 \end{cases} \therefore \forall e \in \mathbb{R} \setminus \{0\} \quad a = \frac{1}{2e^2}, b = \frac{1}{2}$



研究 曲線上の点 (x_1, y_1) における接線の方程式を求め方

$x^2 \rightarrow x_1 x, y^2 \rightarrow y_1 y, x \rightarrow \frac{x_1 + x}{2}, y \rightarrow \frac{y_1 + y}{2}$

を代入すると求められる。